

# On computable numbers, with an application to the Entscheidungsproblem

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July, 2010

# In his seminal 1937 paper, “On computable numbers...”, Alan Turing:

- a) Introduces what is now known as a Turing machine
- b) Provides a concrete construction of a universal Turing machine
- c) Proves the undecidability of the Halting Problem
- d) Provides a convincing argument that any computation performed by a human can also be performed by mechanical means
- e) Solves a famous problem posed by David Hilbert
- f) All of the above

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# In his seminal 1937 paper, “On computable numbers...”, Alan Turing:

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# Turing did not address the Halting Problem

- The paper is concerned with “computable numbers”
- A computable number, by definition, has an infinite decimal expansion
- Therefore, Turing wants his machines to run forever, writing more and more digits on the tape
- So he actually proves the undecidability of the “Prints-infinitely-many-digits Problem”

If a computing machine never writes down more than a finite number of symbols of the first kind, it will be called *circular*. Otherwise it is said to be *circle-free*.

A machine will be circular if it reaches a configuration from which there is no possible move, or if it goes on moving, and possibly printing symbols of the second kind, but cannot print any more symbols of the first kind. The significance of the term “circular” will be explained in § 8.

# The vast majority of textbooks and websites give a misleading account of Turing and the halting problem

The screenshot shows a Windows Internet Explorer browser window with the title 'Turing Halting problem - Bing'. The address bar contains the URL 'http://www.bing.com/search?q=Turing+Halting+problem&form=IE&'. The search bar shows the query 'Turing Halting problem'. Below the search bar, the Bing logo is visible, followed by the search results. The results are categorized under 'Web' and include a list of related searches on the left and a list of all results on the right. The first result is 'Halting problem - Wikipedia, the free encyclopedia', which includes a formal statement and a link to the Wikipedia page. The second result is 'What computers can't do', which discusses the halting problem in the context of Turing machines. The third result is 'Halting Problem -- from Wolfram MathWorld', which explains the determination of whether a Turing machine will halt given a particular input program.

Turing Halting problem - Bing - Windows Internet Explorer

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Turing Halting problem

Web

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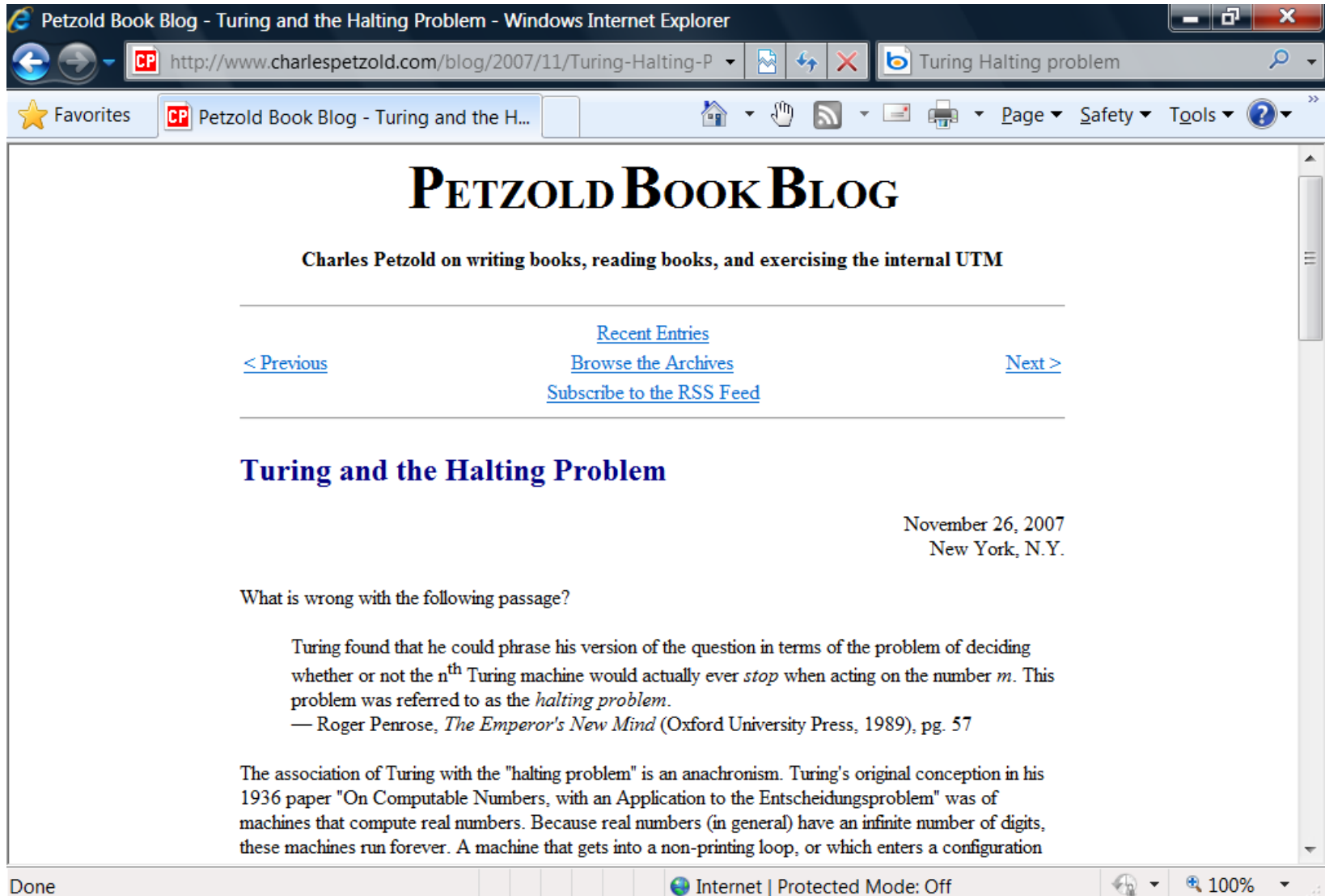
[Halting problem - Wikipedia, the free encyclopedia](#)  
Formal statement · Importance and ... · Sketch of proof · Common pitfalls  
"Theorem 2.2 There exists a **Turing** machine whose **halting problem** is recursively unsolvable. "A related **problem** is the printing **problem** for a simple **Turing** machine Z with respect to a ...  
[en.wikipedia.org/wiki/Halting\\_problem](http://en.wikipedia.org/wiki/Halting_problem) · [Cached page](#)

[What computers can't do](#)  
This **problem** is known as the "**Halting Problem** for **Turing** machines" and was first proved in the 1937 paper in which he introduced his machines. To lead up to that proof, it is ...  
[plus.maths.org/issue5/turing](http://plus.maths.org/issue5/turing) · [Cached page](#)

[Halting Problem -- from Wolfram MathWorld](#)  
The determination of whether a **Turing** machine will come to a halt given a particular input program. The **halting problem** is solvable for machines with less than four states.  
[mathworld.wolfram.com/HaltingProblem.html](http://mathworld.wolfram.com/HaltingProblem.html) · [Cached page](#)

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# I'm not the first person to realize this



The screenshot shows a Windows Internet Explorer browser window. The title bar reads "Petzold Book Blog - Turing and the Halting Problem - Windows Internet Explorer". The address bar shows the URL "http://www.charlespetzold.com/blog/2007/11/Turing-Halting-P". The page content includes the blog title "PETZOLD BOOK BLOG", a subtitle "Charles Petzold on writing books, reading books, and exercising the internal UTM", and navigation links for "Recent Entries", "Browse the Archives", "Subscribe to the RSS Feed", "< Previous", and "Next >". The main article title is "Turing and the Halting Problem", dated "November 26, 2007" from "New York, N.Y.". The article text asks "What is wrong with the following passage?" and quotes Roger Penrose's *The Emperor's New Mind*. The text explains that Turing's original conception in his 1936 paper was of machines that compute real numbers, which run forever.

Petzold Book Blog - Turing and the Halting Problem - Windows Internet Explorer

http://www.charlespetzold.com/blog/2007/11/Turing-Halting-P

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## PETZOLD BOOK BLOG

Charles Petzold on writing books, reading books, and exercising the internal UTM

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### Turing and the Halting Problem

November 26, 2007  
New York, N.Y.

What is wrong with the following passage?

Turing found that he could phrase his version of the question in terms of the problem of deciding whether or not the  $n^{\text{th}}$  Turing machine would actually ever *stop* when acting on the number  $m$ . This problem was referred to as the *halting problem*.

— Roger Penrose, *The Emperor's New Mind* (Oxford University Press, 1989), pg. 57

The association of Turing with the "halting problem" is an anachronism. Turing's original conception in his 1936 paper "On Computable Numbers, with an Application to the Entscheidungsproblem" was of machines that compute real numbers. Because real numbers (in general) have an infinite number of digits, these machines run forever. A machine that gets into a non-printing loop, or which enters a configuration

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# Am I being pedantic?

- Perhaps
- However, the modern definition of *algorithm* is a Turing machine that halts on all inputs.
- The true significance of the Halting Problem's undecidability is that there's no algorithm for identifying other algorithms
- It's interesting that this formulation took years to evolve and did not spring perfectly formed from Turing's 1937 paper.

# Turing's paper is awesome, by the way

- In it, you will find, either explicitly or implicitly:
  - Turing machines, universal Turing machines, nondeterminism, a beautiful argument for the mechanizability of human computation, subroutines, a solution to Hilbert's Entscheidungsproblem, a definition of algorithm, proofs that an immense variety of mathematical calculations can be mechanized