On computable numbers, with an application to the Entscheidungsproblem

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July, 2010

In his seminal 1937 paper, “On computable numbers...”, Alan Turing:

a) Introduces what is now known as a Turing machine
b) Provides a concrete construction of a universal Turing machine
c) Proves the undecidability of the Halting Problem
d) Provides a convincing argument that any computation performed by a human can also be performed by mechanical means
e) Solves a famous problem posed by David Hilbert
f) All of the above

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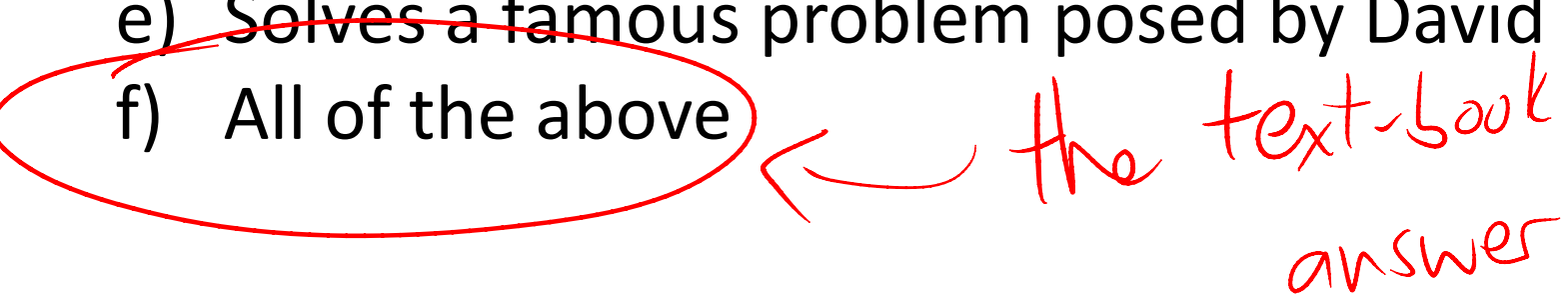

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oops, Turing did not address the Halting Problem

the text-book answer

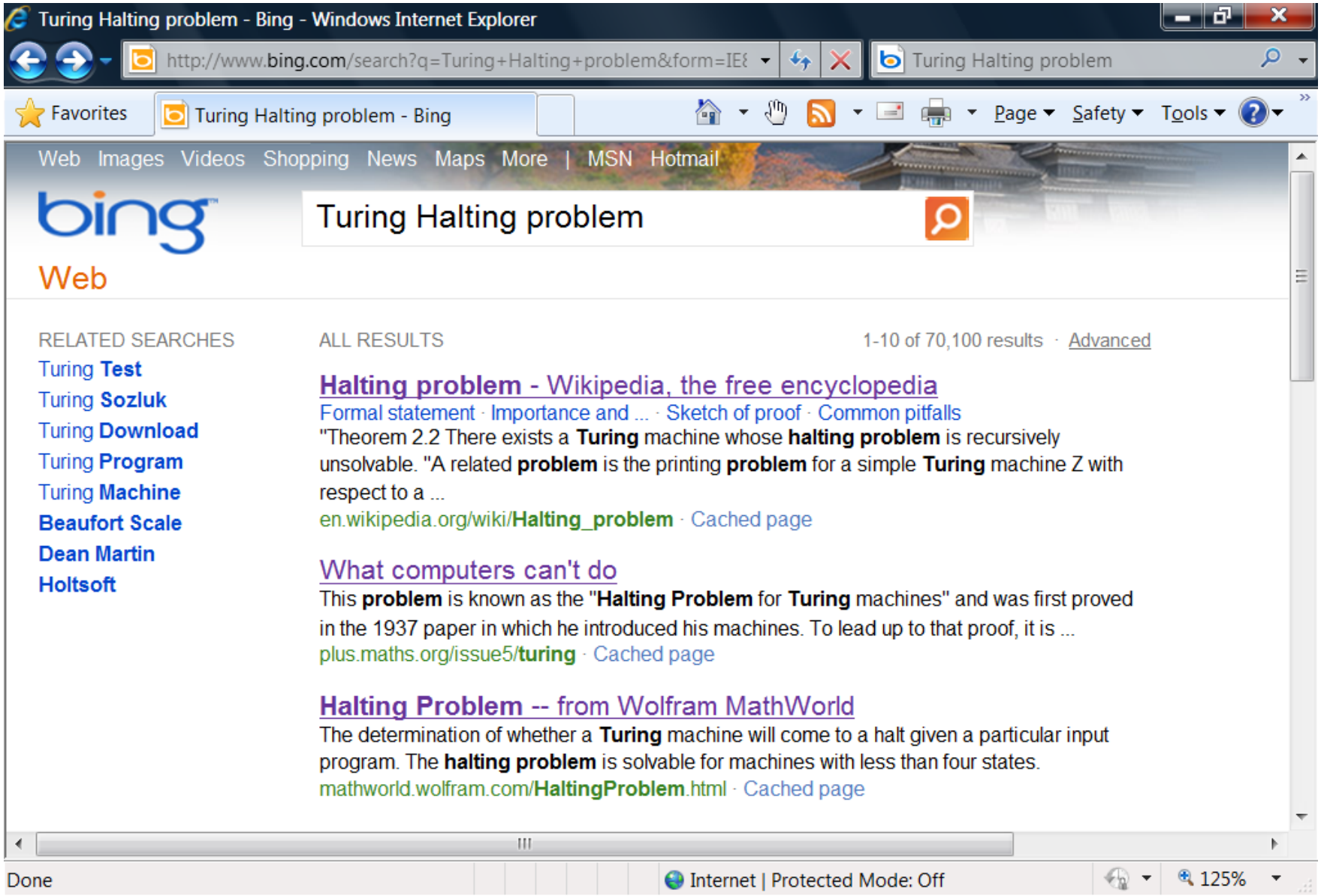
Turing did not address the Halting Problem

- The paper is concerned with “computable numbers”
- A computable number, by definition, has an infinite decimal expansion
- Therefore, Turing wants his machines to run forever, writing more and more digits on the tape
- So he actually proves the undecidability of the “Prints-infinitely-many-digits Problem”

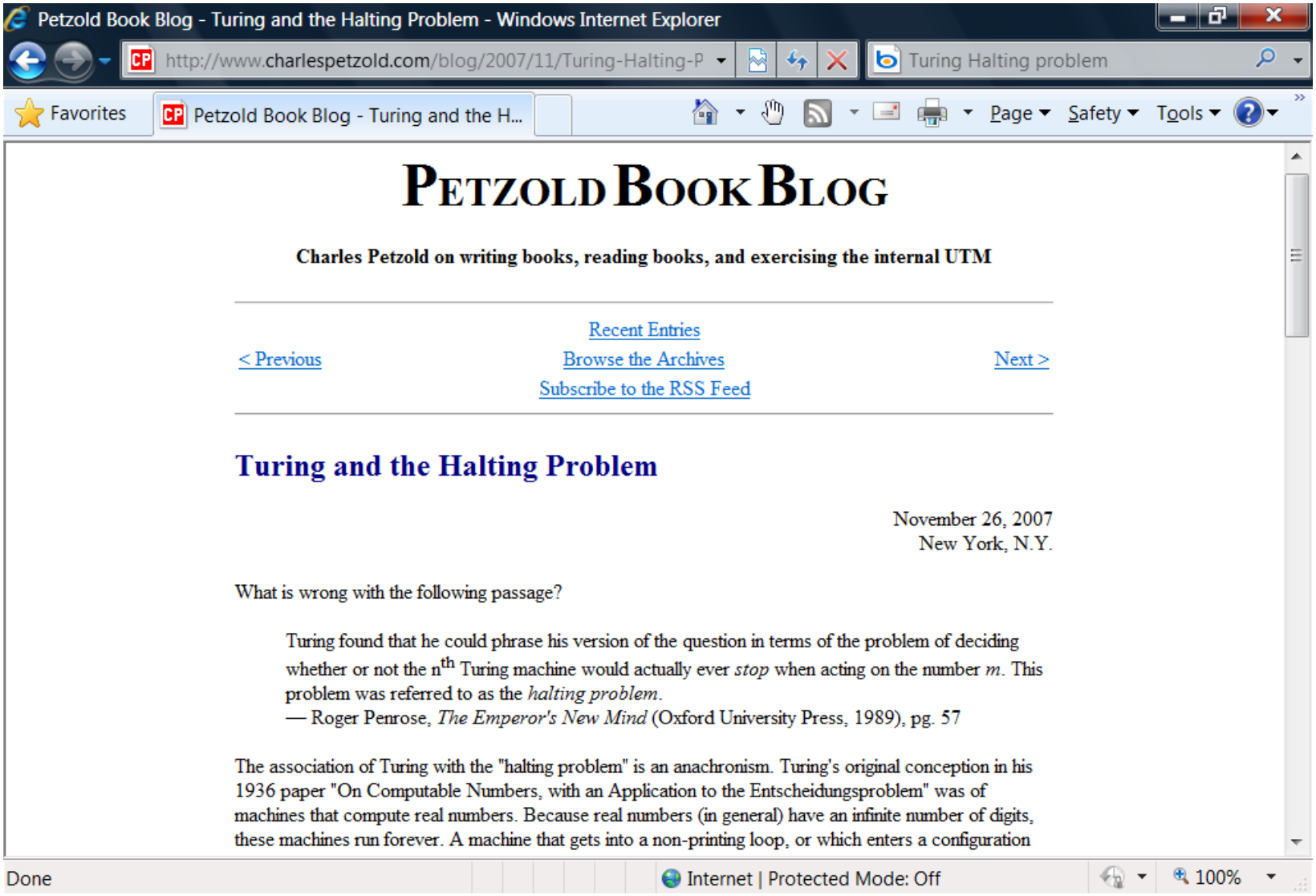
If a computing machine never writes down more than a finite number of symbols of the first kind, it will be called *circular*. Otherwise it is said to be *circle-free*.

A machine will be circular if it reaches a configuration from which there is no possible move, or if it goes on moving, and possibly printing symbols of the second kind, but cannot print any more symbols of the first kind. The significance of the term "circular" will be explained in § 8.

The vast majority of textbooks and websites give a misleading account of Turing and the halting problem



I'm not the first person to realize this



Am I being pedantic?

- Perhaps
- However, the modern definition of *algorithm* is a Turing machine that halts on all inputs.
- The true significance of the Halting Problem's undecidability is that there's no algorithm for identifying other algorithms
- It's interesting that this formulation took years to evolve and did not spring perfectly formed from Turing's 1937 paper.

Turing's paper is awesome, by the way

- In it, you will find, either explicitly or implicitly:
  - Turing machines, universal Turing machines, nondeterminism, a beautiful argument for the mechanizability of human computation, subroutines, a solution to Hilbert's Entscheidungsproblem, a definition of algorithm, proofs that an immense variety of mathematical calculations can be mechanized